Series II Review Part 1

1. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$

$\sin 2x = 2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \dots$

a. $P_5(x) = 2x - \frac{8x^3}{6} + \frac{32x^5}{120}$

$\boxed{P_5(x) = 2x - \frac{4x^3}{3} + \frac{4x^5}{15}}$

b. $\sin \frac{\pi}{7} = \sin 2\left(\frac{\pi}{14}\right)$

$\approx P_5\left(\frac{\pi}{14}\right)$

$\approx \boxed{0.4339}$

2a.

n	0	1	2	3
$f^{(n)}(x)$	$x^{3/2}$	$\frac{3}{2}x^{1/2}$	$\frac{3}{4}x^{-1/2}$	$-\frac{3}{8}x^{-3/2}$
$f^{(n)}(9)$	27	$\frac{9}{2}$	$\frac{1}{4}$	$-\frac{1}{72}$

$P_3(x) = 27 + \frac{9}{2}(x-9) + \frac{\frac{1}{4}(x-9)^2}{2!} + \frac{-\frac{1}{72}(x-9)^3}{3!}$

$\boxed{P_3(x) = 27 + \frac{9}{2}(x-9) + \frac{(x-9)^2}{8} - \frac{(x-9)^3}{432}}$

b. $P_3(10) \approx 31.622685$ $\quad 10^{3/2} \approx 31.6227766$ $\quad$ Remainder: $\boxed{0.0000914}$

3.

n	0	1	2	3
$f^{(n)}(3)$	1	4	6	12

a. $P_3(x) = 1 + 4(x-3) + \frac{6(x-3)^2}{2!} + \frac{12(x-3)^3}{3!}$

$\boxed{P_3(x) = 1 + 4(x-3) + 3(x-3)^2 + 2(x-3)^3}$

b. $f(3.2) \approx P_3(3.2) = \boxed{1.936}$

c. $f'(x) \approx \boxed{Q_2(x) = 4 + 6(x-3) + 6(x-3)^2}$

d. $f'(2.7) \approx Q_2(2.7) = \boxed{2.74}$

4. $x^0 + x + x^2 \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$

a. $\boxed{f(x) = \frac{1}{1-x}, \quad x = -\frac{1}{4}}$ b. $f(-\frac{1}{4}) = \boxed{\frac{4}{5}}$

5. $x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + \frac{(-1)^{n-1} x^n}{n} + \cdots = \ln(1+x)$

a. $\boxed{f(x) = \ln(1+x), \quad x = \frac{2}{3}}$ b. $\ln(1+\frac{2}{3}) \approx \boxed{0.511}$

6. $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots = \sin x$

a. $\boxed{f(x) = \sin x, \quad x = \pi}$ b. $\sin \pi = \boxed{0}$