Series I Review Guide

SEQUENCES

A **sequence** is a list of numbers. Whereas a **series** is a sum of numbers in a sequence.

A sequence $\{a_n\}$ **converges** if $\lim_{n\to\infty} a_n = L \in \mathbb{R}$.

A sequence is **monotonic** if it is always increasing or decreasing. (may be for $n > c$)

$a_{n+1} \geq a_n$ $\qquad$ $a_{n+1} \leq a_n$

If $f(x)$ is continuous and $f(n) = a_n$ $\qquad$ $f'(x) > 0$ $\qquad$ $f'(x) < 0$

A sequence is	bounded from above	bounded from below	bounded	
...if for some $m \in \mathbb{R}$	$a_n \leq m$	$a_n \geq m$	both are true	(may be for $n > c$)

If you can show an appropriate combination of monotonic and bounded, then the sequence converges.

USING CALCULUS WITH SEQUENCES AND SERIES: Change the variable to x !!!

Limits: Try first: rearrange until you can plug in the # For infinite limits: Consider horizontal asymptotes.

l'Hôpital's Rule can only be used when a limit presents the indeterminate form $\frac{0}{0}$ OR $\frac{\infty}{\infty}$.

These require rearrangement before using l'Hôpital's Rule: $\quad 0\cdot\infty \quad \infty-\infty \quad 1^{\infty} \quad \infty^{0}$

Improper Integrals involve an infinite bound.

Full credit work includes...	Correctly written integral (bounds, dx)	Use of a limit	Antiderivative	Operations on bounds	Simplified answer

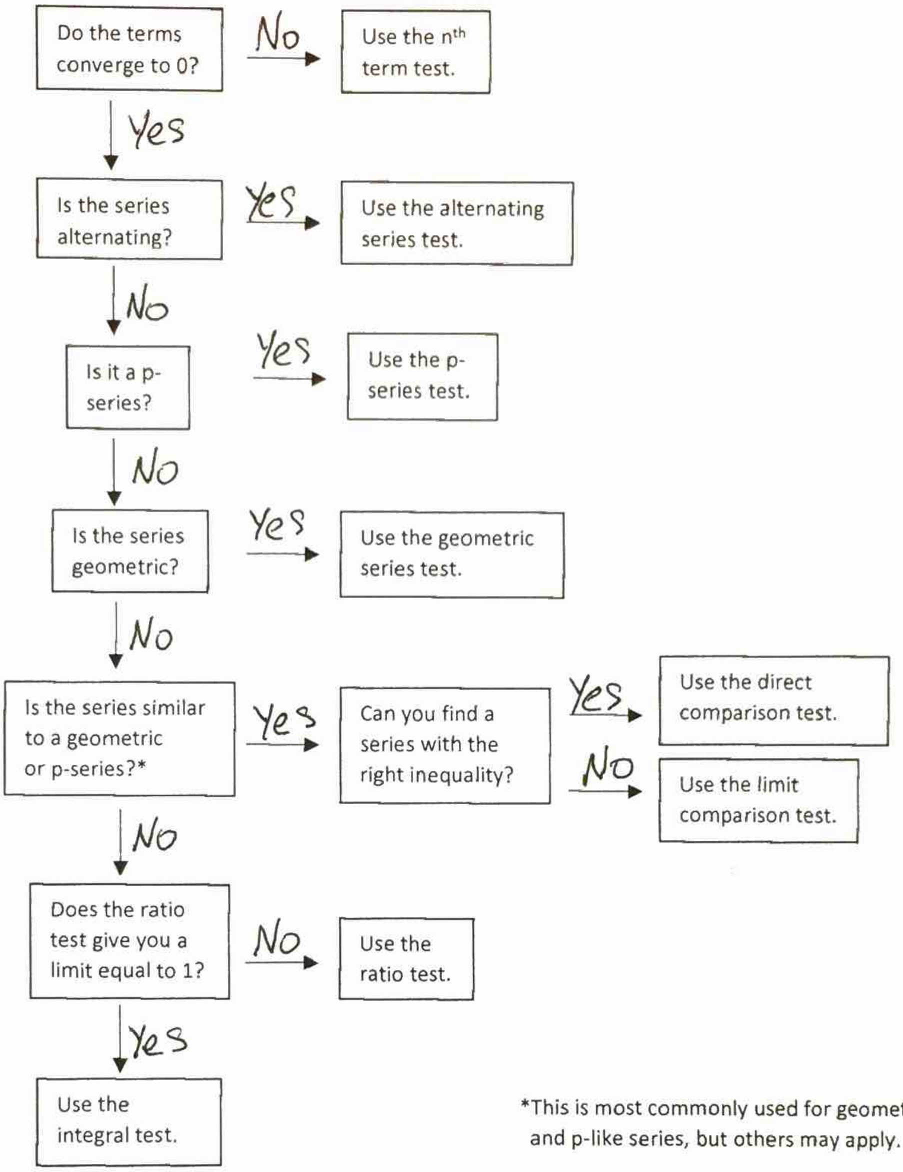
SERIES WE KNOW HOW TO EVALUATE

Type of Series	What It Looks Like	How to Evaluate It
Geometric	$\sum_{k=1}^{\infty} m(r)^k$	Info Book says $S_\infty = \frac{u_1}{1-r}, \lvert r\rvert < 1$ which means $S_\infty = \frac{\text{first term}}{1-r}$
Telescoping	$\sum_{k=1}^{\infty} \frac{c}{(k-s)(k-t)}$	Rewrite with partial fractions $\sum_{k=1}^{\infty}\left(\frac{A}{(k-s)}+\frac{B}{(k-t)}\right)$ Find a pattern and write an expression for S_n. Evaluate $\lim_{n\to\infty} S_n$.

Any series that reveals a pattern with its partial sums.

SERIES CONVERGENCE TESTS

Flowchart



*This is most commonly used for geometric and p-like series, but others may apply.

Scoring Rubrics		
Divergence/nth Term Test +1 Limit is written correctly. +1 Limit is evaluated correctly. +1 Limit is compared to zero. +1 Correct conclusion based on evidence presented. +1 Test named matches work shown.	Alternating Series Test +1 Identify an and an > 0. +1 Limit is written, evaluated, and judged correctly. +1 Monotonic decreasing shown correctly. +1 Correct conclusion based on evidence presented. +1 Test named matches work shown.	Ratio Test +1 Limit is written correctly. +1 Limit is evaluated correctly. +1 Limit is compared to one. +1 Correct conclusion is made based on evidence presented. +1 Test named matches work shown.
Direct Comparison Test +1 Appropriate comparison series (one that works) is chosen and written with correct sigma notation. +1 Convergence/divergence of comparison series is stated with proper justification. +1 The two series are compared with correct inequality. Justification required if not obvious. +1 Correct conclusion based on evidence presented. +1 Test named matches work shown.	Limit Comparison Test +1 Appropriate comparison series (one that works) is chosen and written with correct sigma notation. +1 Convergence/divergence of comparison series is stated with proper justification. +1 The two series are compared with limit written, computed, and judged correctly. +1 Correct conclusion based on evidence presented. +1 Test named matches work shown.	Integral Test +1 Relevant function is continuous, positive, and decreasing (may be for $x > c$). +1 Improper integral is written correctly. +1 Integral is evaluated correctly with work shown. +1 Correct conclusion based on evidence presented. +1 Test named matches work shown.