IB Math HL 2 Formal Calculus and Mathematical Induction Review

1. Given $f(x)=\begin{cases}\sin(x-1)+cx & x\le 1\\ x^2-x+d & x>1\end{cases}$, find constants $c,d\in R$. So that the function is differentiable at $x=1$.

2. Given $f(x)=\begin{cases}-2x-5 & x\le -1\\ x^2-4 & x>-1\end{cases}$, use Rolle's theorem to show that $\frac{df}{dx}=0$ exists on the interval $[-2.5,2]$.

3. Find $\frac{df}{dx}$ where $f(x)=\int_6^{\sin x}\frac{1}{\ln t}dt$. Show your work with proper notations.

4. Given $f(x)=x+\frac{1}{x}$, find all values of c on the given interval [1, 3] such that $f(b)-f(a)=f'(c)\times(b-a)$.

5. Revisit the Formal Calculus quiz.

6. Revisit the Formal Calculus homework.

7. The function f is defined by $f(x)=e^x\sin x$.

a. Show that $f''(x)=2e^x\sin\left(x+\frac{\pi}{2}\right)$.

b. Obtain a similar expression for $f^{(4)}(x)$.

c. Suggest an expression for $f^{(2n)}(x),\ n\in\mathbb{Z}^+$, and prove your conjecture using mathematical induction.

8. (13G#3) From $\sin 2x=2\sin x\cos x$ we observe that $\sin x\cos x=\frac{\sin 2x}{2}=\frac{\sin\left(2^1x\right)}{2^1}$.

a. Prove that i. $\sin x\cos x\cos 2x=\frac{\sin\left(2^2x\right)}{2^2}$ ii. $\sin x\cos x\cos 2x\cos 4x=\frac{\sin\left(2^3x\right)}{2^3}$

b. Assuming the pattern in part a continues, simplify

i. $\sin x\cos x\cos 2x\cos 4x\cos 8x$ ii. $\sin x\cos x\cos 2x\ldots\cos 32x$

c. i. Generalize the results form parts a and b.

ii. Prove your generalization using mathematical induction.

9. Using Mathematical induction, prove $\frac{d^ny}{dx^n}=(-1)^{n-1}\cdot\frac{2(n-3)!}{(1+x)^{n-2}}$ for $n\in\mathbb{Z}, n>2$ if $y=(1+x)^2\ln(1+x)$

IB Math HL 2 Formal Calculus and Mathematical Induction Review ANSWERS

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$P(\_\_)$ is true.

Assume $P(n)$ is true. Then

$P(n+1)$:

So $P(n+1)$ is true. $P(\_\_)$ is true. If $P(n)$ is true then $P(n+1)$ is true. Hence $P(n)$ is true for all n________.

8. (13G#3) From $\sin 2x = 2\sin x\cos x$ we observe that $\sin x\cos x = \frac{\sin 2x}{2\sin x} = \frac{\sin\left(2^1 x\right)}{2^1 \sin x}$.

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