IB Math HL2: Chapter 16 IB Questions

1. (a) Solve the equation $z^3 = -2 + 2\text{i}$, giving your answers in modulus-argument form.

   (b) **Hence** show that one of the solutions is $1 + \text{i}$ when written in Cartesian form.

2. Given that $z$ is the complex number $x + \text{i}y$ and that $|z| + z = 6 - 2\text{i}$, find the value of $x$ and the value of $y$.

3. Given that $\dfrac{z}{z+2} = 2 - \text{i}$, $z \in \mathbb{C}$, find $z$ in the form $a + \text{i}b$.

4. If $z_1 = a + a\sqrt{3}\text{i}$ and $z_2 = 1 - \text{i}$, where $a$ is a real constant, express $z_1$ and $z_2$ in the form $r \operatorname{cis}\theta$, and hence find an expression for $\left(\dfrac{z_1}{z_2}\right)^6$ in terms of $a$ and i.

5. (a) If $w = 2 + 2\text{i}$, find the modulus and argument of $w$.

   (b) Given $z = \cos\left(\dfrac{5\pi}{6}\right) + \text{i}\sin\left(\dfrac{5\pi}{6}\right)$, find in its simplest form $w^4 z^6$.

6. Consider the complex numbers $u = 2 + 3\text{i}$ and $v = 3 + 2\text{i}$.

   (a) Given that $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{10}{w}$, express $w$ in the form $a + b\text{i}$, $a, b \in \mathbb{R}$.

   (b) Find $w^*$ and express it in the form $re^{\text{i}\theta}$.

7. (a) (i) Use the binomial theorem to expand $(\cos\theta + \text{i}\sin\theta)^5$.

   (ii) Hence use De Moivre's theorem to prove

$$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta.$$

   (iii) State a similar expression for $\cos 5\theta$ in terms of $\cos\theta$ and $\sin\theta$.

   Let $z = r(\cos\alpha + \text{i}\sin\alpha)$, where $\alpha$ is measured in degrees, be the solution of $z^5 - 1 = 0$ which has the smallest positive argument.

   (b) Find the value of $r$ and the value of $\alpha$.

   (c) Using (a) (ii) and your answer from (b) show that $16\sin^4\alpha - 20\sin^2\alpha + 5 = 0$.

   (d) Hence express $\sin 72^\circ$ in the form $\dfrac{\sqrt{a + b\sqrt{c}}}{d}$ where $a, b, c, d \in \mathbb{Z}$.