

$$1. L(2+3t, -1+2t, 4+t) \quad P(3, 0, -1) \quad \vec{d} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$\vec{PL} = \begin{pmatrix} 2+3t-3 \\ -1+2t-0 \\ 4+t+1 \end{pmatrix} = \begin{pmatrix} -1+3t \\ -1+2t \\ 5+t \end{pmatrix}$$

$L$  is closest to  $P$  when  $\vec{PL} \cdot \vec{d} = 0$

$$\begin{pmatrix} -1+3t \\ -1+2t \\ 5+t \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = 0$$

$$-3+9t - 2+4t + 5+t = 0 \quad \vec{PL} = \begin{pmatrix} -1 \\ -1 \\ 5 \end{pmatrix}$$

$$\text{Dist from } P \text{ to } L \text{ is } |\vec{PL}| = \sqrt{1+1+25} = \sqrt{27} = \boxed{\sqrt{3\sqrt{3}}}$$

$$3a. l_1: \begin{aligned} x &= -1+2u \\ y &= 10-4u \\ z &= u \end{aligned} \quad l_2: \begin{aligned} x &= -5-2t \\ y &= 3+t \\ z &= 18+3t \end{aligned}$$

$$\frac{x}{-1+2u} = \frac{-5-2t}{10-4u} = \frac{y}{3+t} = \frac{z}{18+3t}$$

$$\begin{aligned} -1+2(18+3t) &= -5-2t \\ 35+6t &= -5-2t \\ 8t &= -40 \\ t &= -5 \rightarrow u = 18-15 = 3 \end{aligned} \quad \begin{array}{l} \text{check} \\ \frac{10-4(3)}{-2} = \frac{-5-2}{-2} \end{array} \quad \stackrel{?}{=} 3-5 \quad \checkmark$$

$$\begin{aligned} x &= -1+2 \cdot 3 = 5 \\ y &= 10-4 \cdot 3 = -2 \\ z &= 3 \end{aligned} \quad \boxed{(5, -2, 3)}$$

$$b. \cos \theta = \frac{\left| \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \right|}{\sqrt{41+16+1} \sqrt{4+1+9}} = \frac{|-4-4+3|}{\sqrt{21} \sqrt{14}} = \frac{5}{\sqrt{21} \sqrt{14}}$$

$$\theta = \boxed{73.0^\circ}$$

4a. Plane contains  $(3, 4, -9)$ ,  $\vec{d}_1 = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$  and  $\vec{d}_2 = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$

$$\vec{n} = \vec{d}_1 \times \vec{d}_2 = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix} \times \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2+8 \\ 12+1 \\ 2-6 \end{pmatrix} = \begin{pmatrix} 10 \\ 13 \\ -4 \end{pmatrix}$$

$$\begin{pmatrix} 10 \\ 13 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 13 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 4 \\ -9 \end{pmatrix}$$

$$10x + 13y - 4z = 30 + 52 + 36$$

$$\boxed{10x + 13y - 4z = 118}$$

b.  $A(7, 2, -1)$  is on  $L_2$  so dist  $L_2$  to plane = dist  $A$  to plane

Line through  $A \perp$  to plane intersects plane when

$$x = 7 + 10t$$

$$10(7+10t) + 13(2+13t) - 4(-1-4t) = 118$$

$$y = 2 + 13t$$

$$70 + 100t + 26 + 169t + 4 + 16t = 118$$

$$z = -1 - 4t$$

$$285t = 18$$

$$t = \frac{18}{285}$$

Dist from  $L_2$  to plane

$$\text{is } \frac{18}{285} |\vec{n}| = \frac{18}{285} \sqrt{100 + 169 + 16} = \frac{18}{285} \sqrt{285} = \boxed{\frac{18}{\sqrt{285}}}$$

5. Show that  $P(3, 1, -2)$  is on plane and

$$\vec{d} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} \text{ is } \perp \text{ to } \vec{n} = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$$

$$P \text{ on plane} \rightarrow 2(3) - (2)(-2) - 2(-2) = 0$$

$$6 - 2 - 4 = 0 \\ 0 = 0 \checkmark$$

$$\begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} = 8 - 2 - 6 = 0 \checkmark$$

$$6. \vec{n}_1 = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \quad \vec{n}_2 = \begin{pmatrix} k \\ -4 \\ 2 \end{pmatrix}$$

a. Parallel  $\rightarrow a\vec{n}_1 = \vec{n}_2$

$$a = -2 \quad -2 \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ 2 \end{pmatrix} \quad \boxed{k = -6}$$

b. Perpendicular  $\rightarrow \vec{n}_1 \cdot \vec{n}_2 = 0$

$$3k - 8 - 2 = 0 \quad \boxed{k = \frac{10}{3}}$$

$$7. \vec{AB} = \begin{pmatrix} -4 \\ -2 \\ 5 \end{pmatrix} \quad \text{Plane contains } A(5, 2, -1), \vec{v} = \begin{pmatrix} -4 \\ -2 \\ 5 \end{pmatrix}, \vec{w} = \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix}$$

$$\vec{n} = \vec{v} \times \vec{w} = \begin{pmatrix} -4 \\ -2 \\ 5 \end{pmatrix} \times \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -4+5 \\ 15+8 \\ 4+6 \end{pmatrix} = \begin{pmatrix} 1 \\ 23 \\ 10 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 23 \\ 10 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 23 \\ 10 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

$$x + 23y + 10z = 3 - 23 + 20 \rightarrow \boxed{x + 23y + 10z = 0}$$

$$8. \begin{bmatrix} 1 & -3 & 1 & 3 \\ 1 & 5 & -2 & 1 \\ 0 & 10 & -6 & k \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & 1 & 3 \\ 0 & -8 & 3 & 2 \\ 0 & 16 & -6 & k \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & 1 & 3 \\ 0 & -8 & 3 & 2 \\ 0 & 0 & 0 & 4+k \end{bmatrix}$$

a. No solution  $\rightarrow k \neq -4$ , infinite solutions  $\rightarrow k = -4$

$$b. \begin{bmatrix} 1 & -3 & 1 & 3 \\ 0 & -8 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad z = t \quad \begin{aligned} -8y + 3t &= 2 & x - 3y + z &= 3 \\ -8y &= 2 - 3t & x - 3\left(-\frac{1}{4} + \frac{3}{8}t\right) + t &= 3 \\ y &= -\frac{1}{4} + \frac{3}{8}t & x + \frac{3}{4} - \frac{9}{8}t + t &= 3 \end{aligned}$$

$$x = \frac{9}{4} + \frac{1}{8}t$$

$$y = -\frac{1}{4} + \frac{3}{8}t$$

$$z = t$$

$$\boxed{\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{9}{4} \\ -\frac{1}{4} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix}} \quad x = \frac{9}{4} + \frac{1}{8}t$$

$$9. \quad 2x + 3y - z = 5 \quad x - y + 2z = k$$

If both planes contain  $5x + 1 = 9 - 5y = -5z$ ,  $\rightarrow \frac{x + \frac{1}{5}}{\frac{1}{5}} = \frac{y - \frac{9}{5}}{-\frac{1}{5}} = \frac{z}{-\frac{1}{5}}$

then  $(-\frac{1}{5}, \frac{9}{5}, 0)$  is on both planes

$$\begin{aligned} x - y + 2z &= k \\ -\frac{1}{5} - \frac{9}{5} + 0 &= k \end{aligned} \quad \boxed{k = -2}$$

2. Reduced Row Form is  $\left[ \begin{array}{ccc} m & 2 & 6 \\ 0 & 4-m^2 & 12-6m \end{array} \right]$ .

a. Unique solution occurs when  $4 - m^2 \neq 0$ , so  $m \neq \pm 2$

b. If  $m = 2$ , then the matrix becomes  $\left[ \begin{array}{ccc} m & 2 & 6 \\ 0 & 0 & 0 \end{array} \right]$  and there are infinitely many solutions.

If  $m = -2$ , then the matrix becomes  $\left[ \begin{array}{ccc} m & 2 & 6 \\ 0 & 0 & 24 \end{array} \right]$  and there are no solutions.